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Modelling Plerion Spectra and their Evolution

Yves A. Gallant

Service d'Astrophysique, C.E.A. Saclay, 91191 Gif-sur-Yvette, France

Eric van der Swaluw

D.I.A.S., 5 Merrion Square, Dublin 2, Ireland

John G. Kirk

MPI für Kernphysik, Postfach 10 39 80, 69029 Heidelberg, Germany

Abraham Achterberg

Astronomical Institute Utrecht, Postbus 80 000, 3508 TA Utrecht, Netherlands

Abstract. We review recent theoretical developments on pulsar winds, their nebulae and relativistic shock acceleration, and show how they illuminate unsolved problems in plerion spectra, in particular the multiple spectral breaks in the Crab and the low-frequency breaks of plerions such as G 21.5–0.9 and 3C 58. Recent work on Fermi acceleration theory at relativistic shocks shows that a particle spectral index of 2.2–2.3, compatible with the X-ray spectra of plerions, results under a wide variety of assumptions. If pulsar winds contain ions as well as electrons and positrons, the mechanism of Hoshino et al. (1992), which yields harder spectra, would operate at lower energies and may explain the flat radio spectral indices of plerions. This scenario implies wind parameters in the Crab compatible with the pulsar wind acceleration model of Lyubarsky & Kirk (2001). Recent hydrodynamical simulations of plerion evolution inside SNR blast waves demonstrate that the passage of the reverse shock rapidly compresses the plerion. Using a simple isobaric model, we investigate the influence of the resulting magnetic field compression and decrease in shock radius on the evolution of the plerion spectrum. We suggest that the passage of the reverse shock may explain the low-frequency breaks in 3C 58 and G 21.5–0.9, as well as the increase in 3C 58's radio flux.

1. X-ray Spectra of Plerions

1.1. Observations and Synchrotron Cooling

Plerions are characterized in X-rays by hard, nonthermal power-law spectra. In the case of the Crab Nebula, where statistics are best, the total integrated spectrum has a best-fit power-law index $\alpha_X = 1.1$ (Toor & Seward 1974) in

energy ($F_\nu \propto \nu^{-\alpha}$, corresponding to a photon index $\Gamma_X \equiv \alpha_X + 1 = 2.1$). Within their larger uncertainties, the X-ray power-law indices of most other plerions appear compatible with this value.

The recent availability of spatially resolved spectra of plerions with *Chandra* and *XMM* reveals spectral steepening towards the edges, with the hardest spectrum at the center having an index around $\Gamma_X \approx 1.6$. The steepening is indicative of synchrotron cooling of a centrally injected hard power-law distribution of electrons and positrons, as is the difference of 0.5 between the central and spatially integrated spectral indices. The corresponding injected particle spectral index is $p = 2.2$, defined by

$$\dot{N}(\gamma)\,\mathrm{d}\gamma \propto \gamma^{-p}\,\mathrm{d}\gamma \tag{1}$$

where $\dot{N}$ is the injection rate and γ the particle Lorentz factor.

1.2. Theory: Fermi Acceleration at Ultra-Relativistic Shocks

This non-thermal population of electrons and positrons is generally assumed to be accelerated at the termination shock of a highly relativistic pulsar wind. Recent investigations of Fermi acceleration at such relativistic shocks (Bednarz & Ostrowski 1998; Gallant & Achterberg 1999; Kirk et al. 2000; Achterberg et al. 2001) have shown that the resulting spectra, in the limit of high Lorentz factors and of a turbulent magnetic field downstream, have power-law indices p in the range 2.2–2.3 for a variety of transport assumptions, compatible with the above inferred value for the injected spectrum.

While further investigations, in particular using 3-D plasma simulations, are needed to confirm that the required levels of magnetic turbulence can be achieved, it seems reasonable to identify the acceleration mechanism for X-ray emitting electrons with Fermi acceleration at the pulsar wind termination shock. Further observational evidence for this scenario comes from gamma-ray burst afterglows, whose spectra can also be explained in terms of Fermi acceleration at the highly relativistic outer blast wave (e.g. Gallant et al. 2000).

2. Radio Spectra of Plerions

2.1. Observations and Broken Power Laws

In the radio domain, plerions are characterized by flat power-law spectral indices, $\alpha_r \approx 0$; the Crab Nebula, with $\alpha_r = 0.3$, has a steeper than average spectrum. The synchrotron loss time of radio-emitting electrons is typically much longer than the age of the plerion, so that the observed spectrum is the injection spectrum integrated over the history of the plerion. Nonetheless, with conventional assumptions about plerion evolution, spectral models assuming a single power-law injection spectrum (Pacini & Salvati 1973; Reynolds & Chevalier 1984) are unable to reproduce the observed spectral index differences $\Delta\alpha > 0.5$ between the radio and X-ray spectra.

Evidence in the Crab Nebula suggests that radio-emitting electron acceleration is still taking place at present, in the central regions near the pulsar wind termination shock (Gallant & Tuffs 2000, 2001; Bietenholz et al. 2001b). Combined with the fact that the nebular spectrum joins smoothly between the

radio and X-ray domains, this suggests the injection of a single population of accelerated particles with a broken power-law spectrum.

2.2. Theory: Resonant Ion Cyclotron Wave Acceleration

Hoshino et al. (1992) have shown that if pulsar winds contain some ions as well as electrons and positrons, an efficient mechanism to accelerate the positrons and electrons is by resonant absorption of ion cyclotron waves collectively emitted at the shock front. Particle-in-cell simulations of this process yielded a range of spectral indices, but these are in general hard, with an average close to the value of $p = 1.6$ needed to explain the Crab Nebula radio spectrum. Moreover, in a quasi-stationary calculation of this process where emission is balanced by absorption (Hoshino & Arons 1991), as might be expected in older systems, the resulting spectral index is $p = 1$, yielding a synchrotron spectral index $\alpha = 0$, exactly the generic plerion radio value.

This resonant ion cyclotron wave mechanism was seen in the simulations of Hoshino et al. (1992) to accelerate positrons and electrons up to a critical energy

$$\gamma_{\rm crit} \sim \frac{m_i}{m_e}\gamma_{\rm sh} \tag{2}$$

at which they resonate with the fundamental ion cyclotron frequency; here m_i/m_e is the mass ratio between ions and electrons, and $\gamma_{\rm sh}$ is the shock Lorentz factor, which is approximately the downstream thermal ion Lorentz factor. From the condition that the electrons must have gyro-radii larger than the shock thickness for Fermi acceleration to operate, it follows that $\gamma_{\rm crit}$ is also the minimum energy for the Fermi mechanism. The picture that emerges for the accelerated particle spectrum is thus of a broken power-law, with a hard spectral index $p_r = 1$–1.6 up to $\gamma_{\rm crit}$, and the steeper Fermi acceleration index $p_X = 2.2$ at higher energies.

3. Cooling Break, Injection Break, and Crab Wind Parameters

If the X-ray spectrum is synchrotron-cooled and the radio one is not, they must be separated by a synchrotron cooling break, with $\Delta\alpha \approx 0.5$, in addition to the aforementioned injection break. The Crab Nebula, whose spectrum is known over most of the intervening frequency range, does indeed show two spectral breaks, one in the far infrared (FIR) around 3×10^{13} Hz, and one in the UV around 10^{16} Hz. (A third break, around 100 keV, will not be discussed here.) Spectral index mapping in the IR, where extinction introduces less uncertainty than in the optical, supports the identification of the FIR break as the synchrotron cooling break (Gallant & Tuffs 2000, 2001). Given the age of the Nebula, this yields a magnetic field $\langle B \rangle \approx 3 \times 10^{-4}$ G, in line with other estimates of the nebular magnetic field.

Identification of the injection break with the UV break then allows inference of the wind Lorentz factor through (2). This yields for the Crab a value of $\gamma_{\rm sh} \sim 10^3$, in stark contrast to the oft-repeated figure of 10^6 from Kennel & Coroniti (1984). It should be emphasized, however, that their model not only does not account for the radio-emitting electrons and positrons in the Nebula, but is incompatible with their originating from the pulsar. If one assumes that

all the radio-emitting $e^{\pm}$ were injected in the Nebula by the pulsar wind, it follows that the time-averaged injection rate of pairs must be $\dot{N}_{\pm} \sim 3 \times 10^{40}\,\mathrm{s}^{-1}$, corresponding with current Crab parameters to a pair multiplicity $\kappa \sim 10^6$. A wind carrying this number of pairs with a Lorentz factor of 10^6 would exceed the Crab spindown power by several orders of magnitude.

There are few theoretical predictions of pulsar wind parameters at the termination shock. One is given by the reconnecting striped wind model of Lyubarsky & Kirk (2001), who derive an asymptotic solution for the wind Lorentz factor as a function of radius. Substituting the Crab pulsar parameters and the above multiplicity of 10^6 yields a wind Lorentz factor at the termination shock $\gamma_{\mathrm{sh}} \approx 2 \times 10^3$, in close agreement with our above observationally derived value. It should be noted, however, that the required high multiplicity is problematic for pair creation models above pulsar polar caps: the recent calculations of Hibschman & Arons (2001) yield $\kappa \sim 10^5$ for the Crab pulsar.

4. Evolution of Plerion Spectra

4.1. Compression by the Reverse Shock

An analytical framework for the spectral evolution of an expanding plerion was presented by Pacini & Salvati (1973), and extended by Reynolds & Chevalier (1984) to the more realistic case of a plerion evolving inside a shell supernova remnant (SNR). Their analyses remain valid in our picture, except for the addition of a break in the injected spectrum. In particular, in the initial phase of supersonic expansion of the plerion, the particle energies and magnetic field both decrease with time as the plerion radius increases, yielding a very steep decline of the radio surface brightness with time, $S_\nu \propto t^{-2.7}$ for $p = 1$ and an approximately constant pulsar spindown luminosity (Reynolds & Chevalier 1984). This helps explain the apparent absence of a radio plerion around some middle-aged pulsars.

This initial phase lasts until the reverse shock from the SNR blast wave reaches the central plerion; at that point, the plerion can be dramatically compressed, as shown by the recent hydrodynamical simulations of van der Swaluw et al. (2001), and subsequently undergoes a slower and initially unsteady subsonic expansion. Compression by the reverse shock increases the magnetic field and particle energies while the radius decreases, which can lead to a significant rebrightening of the plerion (Reynolds & Chevalier 1984). The increased magnetic field can also bring down the frequency of the synchrotron cooling break.

4.2. Low-Frequency Spectral Breaks

A number of plerions such as G 21.5–0.9 and 3C 58 have observed or inferred spectral breaks at comparatively low frequencies ($\sim$100 GHz). Woltjer et al. (1997) have argued that these form a separate class of plerions, and can be explained by a sudden change in the particle or magnetic energy content of the pulsar wind. Here we suggest that the sudden change may occur not in the pulsar wind parameters, but in the plerion confining pressure, as occurs with the reverse shock passage. Assuming that the compression results in a magnetic field in rough pressure balance with the interior of a Sedov blast wave, and that

the compression lasts for a time comparable to the age at which the reverse shock hit, one finds that the synchrotron cooling break can reach the observed low frequencies only in a very dense surrounding medium ($n \sim 30\,\mathrm{cm}^{-3}$).

Such a scenario might be plausible in the case of G 21.5–0.9, where it could explain the small size of the X-ray "halo" around the plerion, interpreted as a non-thermal shell (Slane et al. 2000). The high density would lead to efficient radiative cooling and a rapid shock deceleration, which might explain the lack of observed thermal X-ray emission from this shell. The high extinction to this object ($A_V \sim 10$ from the measured X-ray N_H) would make any optical signature of the shell unobservable. As for 3C 58, its apparent radio brightening with time (Green 1987) may be direct observational evidence that it is currently undergoing compression. This would be consistent with the lack of detected radio expansion of this remnant (Bietenholz et al. 2001a), but the absence of any detected shell emission remains a puzzle.

5. Conclusions

The X-ray spectra of plerions are compatible with Fermi acceleration at ultra-relativistic shocks, which yields a power-law distribution of injected particles with spectral index $p_X = 2.2$–2.3, above a critical energy γ_{crit}. Plerion radio spectra seem compatible with resonant ion cyclotron wave acceleration, yielding a harder power-law index, $p_r = 1$–1.6, and fixing the break energy at $\gamma_{\mathrm{crit}} \sim (m_i/m_e)\gamma_{\mathrm{sh}}$. In the case of the Crab Nebula, identification of the synchrotron cooling break in the FIR and the injection break in the UV inplies a wind Lorentz factor of about 10^3, in sharp contrast with the model of Kennel & Coroniti (1984). This value of the wind Lorentz factor is compatible with the striped wind model of Lyubarsky & Kirk (2001). Finally, compression by the reverse shock might be responsible for the low-frequency breaks observed in G 21.5–0.9 and 3C 58, among others, but this requires a dense surrounding medium.

One prediction of our scenario for particle acceleration is that small-scale features near the wind termination shock, where synchrotron losses have not had time to operate, should reflect the unsteepened injection spectrum, with a single spectral break. The Crab Nebula's wisps should be just such features, and comparison of the frequency of this injection break in the wisps and the Nebula as a whole would then allow a determination of the relative magnetic field values.

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